

P.R.GOV.T.COLLEGE (AUTONOMOUS), KAKINADA

II B.Sc. MATHEMATICS/Semester IV (w.e.f 2017-2018)

Course: Real Analysis

Total Hrs. of Teaching-Learning: 60 @ 4 hr/Week

Total credits: 03

OBJECTIVES:

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.
- This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

Unit 1: Real Number System and Real Sequence

(12 hours)

The algebraic and order properties of $\mathbb{R}$ – Absolute value and Real line – completeness property of $\mathbb{R}$ – applications of supreme property – intervals -Limit point of a set, Existence of limit points. (No questions to be set from this portion)

Sequences and their limits – Range and Boundedness of sequences - Necessary and sufficient condition for convergence of Monotone sequence, limit point of a sequence, Subsequences and the Bolzano Weierstrass theorem - Cauchy sequences – Cauchy's general principle of convergence theorems.

Unit 2: Infinite Series

(12 hours)

Introduction to Infinite Series – convergence of series –Cauchy's general principle of convergence for series – Tests for convergence of nonnegative terms – p- test – limit comparison test – Cauchy's nth root test - De-Alambert's ratio test - alternating series – Liebnitz's test -absolute and conditional convergence.

Unit 3: Limits and Continuity

(12 hours)

Real valued functions – Boundedness of a function - Limit of a Function, One-sided Limits- Right hand and Left Hand Limits - Limits at Infinity - Infinite Limits. (No question to be set)
Continuous Functions- Discontinuity of a Function - Algebra of Continuous Functions – Continuous functions on intervals - Some Properties of Continuity of a function at a point - Uniform Continuity.

Unit 4: Differentiation and Mean Value Theorem

(12 hours)

The Derivability of a function, on an interval, at appoint, Derivability and Continuity of a function - Geometrical meaning of the Derivative - Mean Value Theorems - Rolle's Theorem, Lagranges Mean Value theorem, Cauchy Mean Value theorem.

(12 hours)

Unit 5: Riemann Integration
 Riemann sums, Upper and Lower Riemann integrals, Riemann integral, Riemann Integrable function – Darboux's Theorem - Necessary and sufficient conditions for Riemann integrability – properties of integrable functions – Fundamental Theorem of Integral Calculus – Integral as the limit of a sum – Mean Value Theorems.

Additional Inputs :

1. problems using cauchy's first theorem on limits and cauchy's second theorem on limits.
2. Statement of Maclaurin's theorem and expansions of e^x , $\sin x$, $\cos x$, $\log(1+x)$.

Prescribed book:

- Real Analysis by Rabert & Bartely and D.R.Sherbart, published by John Wiley.

Reference books:

- Elements of Real Analysis by Santhi Nararayan & M.D.Raisinghania, published by S.Chand& Company Pvt. Ltd., New Delhi.
- Course on Real analysis by N.P.Bali-Golden series publications
- A Text Book of Mathematics Semester IV by V.Venkateswarrao & others, published by S.Chand& Company Pvt. Ltd., New Delhi

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

TOPIC	Unit	V.S.A.Q	S.A.Q	E.Q	Marks allotted
MODULE-I	1	2	3	2	33
	2	2	2	1	20
MODULE-II	3	1	1	1	20
	4	1	2	1	33
MODULE-III	5	2	2	1	
TOTAL		8	10	6	

V.S.A.Q = Very short answer questions (1 mark)

S.A.Q = Short answer questions (5 marks)

E.Q = Essay questions (8 marks)

Very short answer questions : $8 \times 1 = 08$

Short answer questions : $6 \times 5 = 30$

Essay questions : $4 \times 8 = 32$

Total Marks

= 70

P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
II YEAR B.SC., DEGREE EXAMINATIONS IV SEMESTER
Mathematics Paper-II : Real Analysis
(Model Paper w.e.f. 2017-2018)

Time: 3 Hrs

Max. Marks:70

PART-I

Answer ALL the questions.

8X1M=8M

1. Define convergence of a sequence.
2. Test the convergence of the sequence $\{a_n\}$, where $a_n = 1 + (-1)^n$.
3. State Cauchy's n^{th} root test.
4. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$.
5. Find $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$ *Define derivative of a function at a point $x=a$.*
6. Give an example of a function which is continuous but not derivable.
7. If $f(x) = x^2$ on $[0,1]$ and $P = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ then find $L(P,f)$.
8. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} [e^{\frac{3}{n}} + e^{\frac{6}{n}} + e^{\frac{9}{n}} + \dots + e^{\frac{3n}{n}}]$

PART -II

Answer any SIX questions choosing at least THREE from each section.

6 X 5M= 30M

SECTION-A

9. Prove that every monotonically increasing sequence which is bounded above converges to its least upper bound.
10. Show that $\lim_{n \rightarrow \infty} \left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \dots + \sqrt{\frac{1}{n^2+n}} \right] = 1$.
11. State and prove Cauchy's general principle of convergence for sequence.
12. Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt{n^3+1} - \sqrt{n^3})$.
13. State and Prove Leibnitz's test.

SECTION-B

14. Examine for continuity the function f defined by $f(x) = |x| + |x-1|$ at 0,1.
15. Show that every derivable function on a closed interval is continuous.
16. Find c of Lagrange's Mean Value theorem for $f(x) = (x-1)(x-2)(x-3)$ on $[0,4]$
17. State and prove fundamental theorem of Integral Calculus
18. Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3 \cos x} dx \leq \frac{\pi^3}{6}$.

PART-III

Answer any **FOUR** questions by choosing at least **ONE** from each section.

4X8=32

SECTION -C

19. Show that the sequence $\{a_n\}$ defined by $a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent.
20. State and Prove D'Alembert's Ratio Test.
21. Test for the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n-1)}{2.4.6.....2n} x^{n-1}$ ($x > 0$).

SECTION-D

22. Prove that every continuous function is bounded and attains its bounds.
23. State and prove Rolle's theorem
24. Prove that $f(x) = \sin x$ is integrable on $[0, \frac{\pi}{2}]$ and $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$
